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DEVELOPMENT OF A RESEARCH PLAN
FOR THE IMPROVEMENT OF AERODYNAMIC MODELS
FOR ANALYSIS OF BALLISTIC RANGE DATA

by

Robert W. Courter
Department of Mechanical Engineering
Louisiana State University

FINAL REPORT

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A study is made of the relationship between aerodynamic models and parameter estimation methods to determine a feasible plan for improving the aerodynamic models which are used for ballistic range data analysis at Eglin AFB. A plan is presented which utilizes a computer graphics terminal to process simulation runs and provide comparisons with experimental data and theoretical data fits. Included in the report are algorithms for performing system simulation and for accounting for mass-offset effects on the motion of the test item.

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PREFACE

The work documented in this report was performed under U.S. Air Force Grant Number AFOSR-78-3489. The work was performed at the Louisiana State University in Baton Rouge, Louisiana, in the Department of Mechanical Engineering. Dr. Robert W. Courter is the Principal Investigator.

Acknowledgement is made to Mr. Paul Core and Mr. Ameziane Harhad for their help.

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NOMENCLATURE

A	Reference Area
B	Asymmetries Euler angle Matrix
b_{ij}	Matrix elements of B
C	Inertia transformation matrix
c_{ij}	Matrix elements of C
C_{l_β}	$\frac{\partial(L/\bar{q}A\ell)}{\partial\beta}$
$C_{m_{\dot\alpha}}$	$\frac{\partial(M/\bar{q}A\ell)}{\partial(\dot\alpha\ell/2V)}$
C_{n_β}	$\frac{\partial(N/\bar{q}A\ell)}{\partial\beta}$
C_{x_α}	$\frac{\partial(F_x/\bar{q}A)}{\partial\alpha}$
$C_{x_{\dot\alpha}}$	$\frac{\partial(F_x/\bar{q}A)}{\partial(\dot\alpha\ell/2V)}$
C_{x_u}	$\frac{\partial(F_x/\bar{q}A)}{\partial(u/V)}$
C_{y_β}	$\frac{\partial(F_y/\bar{q}A)}{\partial\beta}$
C_{y_p}	$\frac{\partial(F_y/\bar{q}A)}{\partial(p\ell/2V)}$
C_{y_r}	$\frac{\partial(F_y/\bar{q}A)}{\partial(r\ell/2V)}$
C_{z_α}	$\frac{\partial(F_z/\bar{q}A)}{\partial\alpha}$
$C_{z_{\dot\alpha}}$	$\frac{\partial(F_z/\bar{q}A)}{\partial(\dot\alpha\ell/2V)}$
C_{z_u}	$\frac{\partial(F_z/\bar{q}A)}{\partial(u/V)}$
d	Body diameter
F_x	Body force along x-axis

F_y	Body force along y-axis
F_z	Body force along z-axis
f	Axis displacement from c.g. along x_1 -direction
G_{ij}	Elements of axis displacement matrix
g	Axis displacement from c.g. along y_1 -direction
H	Angular momentum
h	Axis displacement from c.g. along z_1 -direction
I_{ij}	Inertia components
ℓ	Reference length
L	Rolling Moment
M	Pitching Moment
$\mathcal{M}$	Mach number
$\mathfrak{M}$	Moment cross product term
N	Yawing moment
P	Angular velocity vector
p	Roll rate
q	Pitch rate
$\bar{q}$	Dynamic pressure, $\frac{1}{2}\rho V_\infty^2$
r	Yaw rate
t	Time
U	Translational velocity vector
u	Velocity in x-direction
V_∞	Total velocity, $\sqrt{u^2+v^2+w^2}$
$\mathcal{V}_i$	Velocity cross product term
v	Velocity in y-direction
w	Velocity in z-direction

X	Force vector
$\bar{X}$	Axis displacement vector
x	Coordinate
y	Coordinate
z	Coordinate
α	Angle of attack, $\sin^{-1}(w/V_\infty)$
$\bar{\alpha}$	Total angle of attack, $\sin^{-1}(\sqrt{v^2+w^2}/V_\infty)$
β	Angle of sideslip, $\sin^{-1}(v/V_\infty)$
δ	Asymmetry yaw angle
ϵ	Asymmetry pitch angle
ϵ_{ijk}	Permutation tensor
ρ	Mass density of atmosphere
σ	Asymmetry roll angle
ψ	Projectile yaw angle
θ	Projectile pitch angle
ϕ	Projectile roll angle
ω	Angular velocity vector
λ	Angle of attack function

Subscripts

$(\ )_o$	Reference condition
$(\ )_1$	c.g. axes
$(\ )_2$	other axes

SECTION I

INTRODUCTION

1. Subject Matter

The determination of the aerodynamic characteristics of missiles and projectiles from free-flight ballistic range data is dependent on three main factors: (1) accurate ballistic range orientation and position data, (2) appropriate analytical aerodynamic models for use in parameter estimation programs which adequately account for configuration and flow field characteristics, and (3) estimation algorithms which provide reliable estimates of the aerodynamic coefficients using the given data and model. This report deals with a study of methods to improve the accuracy of the second of these important factors.

The emphasis of the study is placed on the development of a model description procedure which properly correlates flight item configuration and kinematics with major flow field characteristics. The motivation behind this particular approach is to provide for the selection of a model form on the basis of configuration and expected flow field parameters.

2. Historical Background

Since the least squares extraction method of Chapman and Kirk (Reference 1) came into use in the analysis of ballistic range data, increasingly complex forms of aerodynamic models have been employed in various parameter estimation programs. All of these models have been based on the series expansion of aerodynamic forces and moments in terms of various kinematic variables. The coefficients in these expansions, usually called stability derivatives, are then determined by the parameter estimation process. The recent enlargement of the viable envelope for ballistic testing brought about by improvements in experimental and analytical methods has put considerable emphasis on the need for improved aerodynamic models as well.

Two indirect methods for model structuring have been recently reported. In Reference 2 the Estimation Before Modeling (EBM) method is described. This technique has the particular advantage that a state-dependent model need not be selected prior to state identification. Thus, total force and moment values are determined, and particular model forms for comparison with data from other sources can be determined over very restricted regions of the parameter space (e.g., over a specified range of angles of attack). Reference 3 describes recent advances in model identification methods. The general technique here is to analytically describe an aerodynamic model which is very general in terms of independent variables which can be used to describe the forces and moments. Then

the particular model is selected based on comparisons with range data. This method thus determines the model form and the coefficient values through interaction between the model form and the extraction algorithm. The method allows for change in model form when the physical behavior of the system within a given range of independent variables requires that such a change be made. In this way the method indirectly incorporates a correlation with the physical flow field behavior.

Preliminary studies of aerodynamic models were performed by the Principal Investigator during a USAF/ASEE Summer Research Fellowship at Eglin AFB. These studies, which concentrated on determining existing analysis methods and initiating mass-offset studies and investigations into model form are presented in Reference 4. The results of that investigation were sufficiently encouraging to lead to a more detailed formulation of the model selection process. The present report gives the details of the investigation procedure which was developed.

3. Scope

The remainder of this report is devoted to an explanation of the studies which led to the formulation of a research plan for the improvement of aerodynamic models for use in ballistic range data analysis. These studies consist of three major phases: (1) review of current analytical methods in the field of parameter estimation to determine model-dependency, particularly with regard to analytical form, (2) establishment of a data base from which aerodynamic model inferences may be made and (3) analytical determination of mass-offset effects on dynamic motion through derivation of various transformation relations. The development of the analytical tools required to implement these model studies is also carried out.

Section II contains a description of the analytical schemes which are currently being used by various agencies to determine the aerodynamics of flight items through analysis of free-flight data. The emphasis of this discussion is concerned with the role of the aerodynamic model in the extraction process.

Section III identifies the type and source of aerodynamic data for three general classes of flight item. These are: the axisymmetric body alone, the axisymmetric body with tail surfaces and the axisymmetric body with wing and tail surfaces. Special attention is paid to non-linear aerodynamic characteristics, since the current difficulties with model selection are predominantly because of these non-linear terms. Also included in this section is a discussion of the adaptability of several different model forms in the coefficient extraction process.

Section IV provides the details of the general coordinate, inertia and aerodynamic coefficient transformations necessary to effect a mass-offset analysis of a free-flight test item. Sensitivity equations are also derived which indicate the degree to which a basic parameter is affected by an axis offset.

Section V outlines a specific research plan incorporating the results of this study. The purpose of this plan is to provide a systematic technique for evaluating a variety of aerodynamic model forms. Block diagrams of the model evaluation procedure are presented together with proposed techniques for providing a correlation between aerodynamic model formulation and identifiable flow field characteristics.

Section VI is a summary of the information in this report. Important conclusions and special features of the study are presented here.

In Appendix A the essential features of the mass-offset transformations are presented. In Appendix B program listings for simulation and transformation calculations are presented.

SECTION II

PARAMETER ESTIMATION METHODS

1. Introduction

There are four methods of parameter estimation which have direct application to the free-flight data analysis problem and which appear to be at the forefront according to recent references. The details of these methods are important in the present work only with regard to the role which the aerodynamic model plays in the computations. Particular attention is paid to the effect of various model forms on the estimation calculations.

The basic methods under consideration here are: the extended Kalman filter (EKF), maximum likelihood method (MLM), estimation before modeling (EBM) and model structure determination (MSD). One other method was given careful consideration during the evaluation period. The square-root variable metric technique described in Reference 5 was ruled out as a possible method for ballistic analysis because the numerical derivative computations require more data for accurate implementation than is usually available from ballistic tests.

There is some evidence to suggest that the form of the aerodynamic model affects the accuracy of the extraction computations if an unduly large number of coefficients are being extracted. This tendency precludes representation of the aerodynamic terms in very general terms. It is important to be able to identify important physical effects which are expected in the flight envelope and to reflect these in the model so that the number of cefficients being sought can be as small as possible. Based on evidence from numerous applications of parameter estimation methods to ballistic data analysis at Eglin AFB, the order in which coefficients are extracted also affects the accuracy of the outcome. This factor was considered in the study of the extraction methods discussed here.

Finally, it should be emphasized that the purpose of this section is not to select an extraction algorithm, but rather to determine any special features which a model should have to be effective in a particular method. Currently, the maximum likelihood method is being used for data analysis at the Aeroballistic Research Facility at Eglin AFB. Further model studies at that installation will be made with that extraction algorithm.

2. Extended Kalman Filter

The Kalman filter theory determines the state of a system by utilizing measurements of some or all of the system state variables to make estimates of the system condition at the next interval. Actually, the original formulation applied only to linear systems, but the EKF method can be applied to highly non-linear systems. This method of analysis is a special case of the more general maximum likelihood method.

The application of this method to the free-flight trajectory analysis problem follows the same fundamental steps as the original Chapman-Kirk method. That is, an analytical trajectory is computed using a specified aerodynamic model. This analytical model is then compared with discrete trajectory parameters measured in the ballistic range. A least squares minimization is then performed yielding a set of aerodynamic coefficients which provides the best analytical fit to each experimental data point in the trajectory. The basic difference between the two methods lies in the minimization algorithm. The EKF algorithm permits solutions of the coupled dynamic equations while the Chapman-Kirk method requires that the translational equations be solved independently from the rotation equations. Both methods, however, commonly employ polynomial expansions to represent the aerodynamic terms in the equations of motion. This model format is common to most dynamic performance, stability and control studies (see Section V), and it simplifies correlation with experimental results from other forms of test.

There is no inherent restriction in the EKF method on the model form as long as the analytical formulation can be put in terms of the state variables. Some difficulties have been reported, however, in the operation of the algorithm if initialization is not carefully achieved. The EKF performance is very sensitive to the initialization of parameters, so a model which cannot be well evaluated initially may lead to a poor filter performance even though the model itself may be very good.

3. Maximum Likelihood

This method is a more general modification of the basic least squares method of Chapman and Kirk which utilizes a different optimization scheme to achieve coefficient extractions. In that method the solutions were achieved for the uncoupled equations of motion because the scales of the deviation matrices for translational and rotational motions were different, precluding the coupled analysis. The maximum likelihood algorithm applies weighting factors to the traslational and rotational deviation functions so that the best fit of the analytical model to the experimental data can be achieved by maximizing a likelihood function (that the estimate and the actual data coincide). The aerodynamic model plays the same role in this model as it does in the Chapman-Kirk and EKF methods. There seems to be nothing inherent in the method which would rule out any model form with undetermined coefficients for describing the aerodynamic forces and moments. However, it is essential that the formulation be posed in such a way that the equations of motion can be analytically differentiated with respect to the coefficients so that the sensitivity equations used in the statistical analysis can be determined.

This technique is currently used at Eglin AFB to determine aerodynamic coefficients from ballistic data. One disadvantage in doing model studies with this or the method previously discussed is that analytical expressions are required for the evaluation of sensitivity parameters. Thus an internal change in the computer code must be made with each model change. The form of the model could, thus, affect its implementation into the extraction program because of coding or storage difficulties, but the quality of the estimation should not be compromised because of this difficulty.

4. Estimation Before Modeling

This technique is quite different from those previously described in that an aerodynamic model is not required prior to the first estimation calculation. Furthermore, the results of this analysis are in the time, rather that state, domain, so the total aerodynamic forces and moments are determined only as time functions.

It is assumed that the motion of the flight item is described by a system of non-linear differential equations (the equations of motion) in which the state-dependence of certain terms (e.g., aerodynamic forces and moments) is unknown. These unknown terms are replaced by special spline time series with unknown coefficients. These coefficients are then determined by conventional means (e.g., maximum likelihood) through comparison with experimental data. Once the time domain solutions become available, the total aerodynamics are known. That is, the forces along each axis, the moments about each axis and any auxiliary state-dependent functions, such as control forces and moments, become completely determined in the time domain. Correlation of these results with the state-time relationship provided by the equations of motion yields the values of the total aerodynamic characteristics for any given value of a fundamental state variable or for a value of a state-variable-determined quantity (such as angle of attack or Mach number).

This technique provides the total aerodynamic force and moment as a function of time. However, no detailed information regarding the contributing parts of the total is provided. Thus, the resulting total provides little direct information regarding the effects of variation of parameters on the total result. This makes the result impossible to check directly with other test data (e.g., wind tunnel data). In addition, the total force and moment functions are totally dependent on the degree of fit achieved by the estimation process.

One facet of this application which is appealing as a possible modeling technique is the use of splines for function approximation in the analysis. The details of various spline functions, as outlined om Reference 6, indicate that a complex function can be described with only a few spline regions provided the approximated functions are smooth. It is felt that this approach is worth additional study with regard to applications to conventional stability derivative functions which seem not to be adequately described by standard polynomial functions.

5. Model Structure Determination

A very recent innovation in the broad field of parameter estimation and system performance evaluation is the application of statistical data comparisons to the determination of the basic structure of the models which influence system behavior. As applied to ballistic range data analysis this method, called model structure determination (MSD), would repetitively attempt to match experimental data with a

variety of models for the aerodynamic forces and moments. The basic extraction algorithms could be identical to those described in parts 2 and 3 earlier, but the analysis and statistical testing goes beyond that. By this technique any number of model forms, each with its own set of undetermined coefficients, can be included in the search for a model or group of models which will provide the best simulation of the experimental trajectory. Such an approach allows for the simulation of rapidly fluctuating aerodynamic terms which may arise from such things as vortex roll-up, shock stall or wake impingement.

The general model form required in this analysis is one for which appropriate sensitivity relations can be derived by differentiation with respect to the parameters. This obviously makes the choice very wide, ranging from a simple algebraic polynomial to a Fourier series. The number of different model forms and the generality of each is limited by the computational constraints of time and storage. In any given case the model forms are selected based on the physical phenomena they are expected to describe.

6. Summary

The studies which have been discussed here have been based completely on information available in the open literature and on actual computational results which have been achieved by the Ballistics Branch at Eglin AFB, Florida. The Principal Investigator has only used the Chapman-Kirk and maximum likelihood methods with polynomial aerodynamic models as they are formulated in the operational programs at Eglin AFB. Nevertheless, some conclusions of a general nature can be concluded from the present study of model/extraction method interaction.

For a great many flight situations the aerodynamics can be accurately represented by conventional algebraic polynomial expansions. In all of the methods described the polynomial can be conveniently incorporated. Furthermore, the "stability derivative" format is easily compared with results from other types of tests. However, there are some physical phenomena which cannot be simulated with this model. These include forces and moments in regimes where neither even nor odd function characteristics are evident because of complex interaction, in flow fields containing significant vortex growth and for cases in which violent maneuvers are encountered because of surface stall. Other series type formulations could be used to simulate these cases. For example, transcendental series, splines or any of the special form series such as Fourier or Legendre could be used.

It appears imperative in the continuation of these model studies that unusual physical phenomena which are observed to occur in the flight dynamics of various configurations must be correlated to specific model types so that a systematic model development program can evolve. Of course, such a study requires experimental data on a variety of configurations and flight conditions. The next section describes initial investigations in this category.

SECTION III

AERODYNAMIC DATA COLLECTION

1. Introduction

In order to develop a systematic approach to the model formulation task it is necessary to establish some development guidelines. One of these guidelines involves configuration selection. The motivation behind establishing certain classes of configuration types is to simplify the task of identifying those parts of the aerodynamic model which are strongly configuration-dependent.

The present research plan is predicated on defining models for three classes of flight item. These are: bodies of revolution without control surfaces, bodies of revolution with tail fins only and bodies of revolution with tail and wing surfaces. Of course, a large volume of wind tunnel data are available in the technical literature of the past thirty years on configurations of all types, including the three classes of interest here. The emphasis in seeking data, however, has been placed on establishing modeling criteria in the non-linear regions of the flight profile such as at high angle of incidence or in regions where induced effects are important. Typical data sources and types for each of the three configuration classes will now be given.

2. Body Alone

Probably more data collection has been done on bodies of revolution than on any other class of flight item. The data ranges from basic, drag and moment data to induced forces and moments for spinning bodies. The computations to determine the linearized characteristics (low angle of incidence) at all speeds are outlined in such basic sources as References 7,8 and 9. In addition NACA and NASA have published numerous reports of wind tunnel tests of a variety of body configurations at all flight speeds. As long as the data are at moderate angle of attack they can be adequately modeled by conventional polynomials. Indeed such characteristics as lift, drag and moment variations with angle of attack and Mach number hold no surprises.

Magnus forces and moments and other incidence and rotation induced effects are not so well known. Reference 10 contains data on cone-cylinder and ogive-cylinder which would be difficult to fit with a regular polynomial at high angles of attack. However, the data do appear to be amenable to polynomial fitting on a piecewise basis. Reference 11 contains fundamental pressure data for six nose-cylinder-boattail bodies of revolution for angles of attack to 60° at supersonic Mach numbers. Integration of these data should yield sufficient information to model some of the high angle non-linearities evident in force and moment data. In fact, when used in conjunction with the distributed vortex model of Reference 12, a picture of vortex effect modeling should evolve.

A very important part of the data base is a large collection of ballistic data accumulated by the AFATL staff at Eglin AFB. A variety of projectile configurations have been tested ballistically and in the wind tunnel to provide a very good set of related data. Most of these configurations are of the projectile type, however, rather than being of a shape usually associated with a missile or airplane. Also, they are necessarily spin stabilized, so the aerodynamic behavior at high angle of attack cannot simulate that for a non-spinning body. In light of the experience of many years of projectile testing, it is no surprise that the ballistic data can be fit with very good accuracy by the aerodynamic models currently in use (Reference 13). In fact, a recent investigation (Reference 14) indicates that accurate estimates can be achieved even when the projectile contains a moving internal component.

3. Body-Tail Fin

The most common type of guided missile is the finned body. There are many configurations of finned missiles which consist of a body of revolution with tail surfaces. These can be spin or fin stabilized. The fin stabilized missile introduces two classes of aerodynamic non-linearity that are not usually encountered in the unfinned geometries. These are fin-body interference (and vise versa) and body vortex shedding at incidence. In addition, the fin geometry, position on the body and body geometry all influence the aerodynamics. Fortunately there has been a considerable amount of testing associated with finned missiles. A configuration known as the Basic Finner, which consists of a cone-cylinder body with four rectangular planform tail fins in horizontal and vertical planes, has been tested extensively in free flight and in the wind tunnel (e.g., see Reference 15). Supporting analytical work for configurations at low angles of attack is also available from a variety of sources such as Reference 16.

Availability of data for these configurations at high angle of incidence is less secure. A recent report of a ballistic research project at Eglin AFB is encouraging (Reference 17). Extensive wind tunnel and free flight tests have been performed on an ogive-cylinder body with swept leading edge tail fins. Angles of attack on some of the tests are near 30° and distance non-linearities are observed. It appears that the aerodynamic models could be extended with a matched polynomial (like a spline) to incorporate these areas of interest. Visual studies of the flow have not been made to determine the probable cause for the high-angle behavior. Similar studies have been performed by NASA (Reference 18) on a finned body with much lower fineness ratio and swept vane-type fins.

A computer search of government documents has failed to turn up test data for tail-finned configurations at high incidence published since 1970. Most of the basic testing on these configurations was probably performed in the 1950's and is available in the government archives. On the other hand, the fin stabilized missiles are not intended to operate in the high incidence regime and extensive testing in that regime may never have been authorized.

The model forms which are currently being employed for studying conventional configurations at low angle of incidence appear to be adequate. These involve representation of the forces and moments as polynomial functions of Mach number, sine of the total incidence angle and the aerodynamic roll angle.

4. Body-Tail-Wing

The advent of new high performance, high maneuverability aircraft during this decade has necessitated the development of new offensive and defensive missile weaponry. Missiles now in the inventory frequently fly in a high-g, high incidence angle environment. These configurations represent the ultimate challenge to ballistic testing and data analysis. It is with these configurations that the greatest innovations in model formulation will be necessary. Typical of the data with severe non-linearities which result from such configurations are the test results shown in Reference 19. This particular configuration experiences severe non-linear excursions in weathervane and lateral stability at angle of attack. Proper modeling of just this one effect would require a radical departure from the conventional polynomial representation. Similar non-linear behavior in the longitudinal mode is reported in Reference 20 for recent maneuvering missile configurations. It may well be impossible to isolate each physical effect which is causing the extreme non-linearities because the configurations are complex and the flow fields very difficult to analyze.

SECTION IV

MASS-OFFSET CALCULATIONS

1. Introduction

In ballistic testing it is important to know the exact center of gravity (c.g.) location of the model because the free-flight behavior will be strongly influenced by this location. Model measurements prior to testing determine the longitudinal position of the c.g. and the moments of inertia of the body about geometrical axes of symmetry through this longitudinal c.g. position. The data reduction of the test flight is then predicated on the c.g. lying on the axis of symmetry. If the c.g. is displaced from this axis, it is necessary to analyze the dynamics based on that displaced position. Such an analysis requires that the corresponding inertias and aerodynamic coefficients also be transformed to the new position so that the Newtonian equations of motion will apply.

The fundamental concepts of these transformations were given in a very general form in Reference 4. A summary of this work plus some newer developments on coefficients and sensitivities is given in the following paragraphs.

2. Coordinate Transformations

Figure 1 illustrates the basic coordinate relationships. Using the angular and translational displacements as parameters, the coordinates are transformed by

$$\{X_2\} = [B]\{X_1 - \bar{X}\} \tag{IV-1a}$$

$$\{X_1\} = [B]^T\{X_2\} + \{\bar{X}\} \tag{IV-1b}$$

The matrix elements for these and the following transformations are shown in Appendix A.

3. Inertia Transformations

The inertia tranformations between c.g. axes and axes of symmetry are given by

$$\{I_2\} = [C]\{I_1\} + \{G\} \tag{IV-2a}$$

4. Velocity Transformations

The translational and rotational velocity components tranform as follows:

$$\{U_2\} = [B]\{U_1 - \mathcal{V}\} \tag{IV-3a}$$

$$\{U_1\} = [B]^T\{U_2\} + \{\mathcal{V}'\} \tag{IV-3b}$$

$$\{P_2\} = [B]\{P_1\} \tag{IV-4a}$$

$$\{P_1\} = [B]^T\{P_2\} \tag{IV-4b}$$

5. Force and Moment Transformations

The total forces and moments can be transformed according to the following relations:

$$\{F_2\} = [B]\{F_1\} \tag{IV-5a}$$

$$\{F_1\} = [B]^T\{F_2\} \tag{IV-5b}$$

$$\{M_2\} = [B]\{M_1 - \mathcal{M}\} \tag{IV-6a}$$

$$\{M_1\} = [B]^T\{M_2\} + \{\mathcal{M}'\} \tag{IV-6b}$$

6. Aerodynamic Coefficient Transformations

In order to apply the Newtonian equations of motion, the axis system must be located at the c.g., and the forces and moments must be defined with respect to these axes. In the usual case the c.g. is on the axis of symmetry, and the simulation can proceed normally. In the case of a flight item with mass-offset, however, the axes are displaced, and corrections must be made. These corrections include aerodynamic coefficient transformations.

For most flight items the axis displacements will be very small, and approximate corrections can be made. However, in this report a general formulation is made without any small displacement approximations. A typical derivation of one of the aerodynamic coefficients is shown in the following paragraph. Other coefficients can be derived in a similar manner. A listing of the key coefficients is shown in Appendix A.

Consider the case where characteristics known with respect to axes (x_2, y_2, z_2); an aerodynamic reference axes set, are to be transformed to the center of gravity axes set (x_1, y_1, z_1). Typically,

$$F_{x_{\alpha_1}} = \frac{\partial F_{x_1}}{\partial \alpha_1} = \frac{\partial F_{x_1}}{\partial w_1}\frac{\partial w_1}{\partial \alpha_1}$$

$$= \frac{\partial}{\partial w_1}\left(b_{11} F_{x_2} + b_{21} F_{y_2} + b_{31} F_{z_2}\right)\frac{\partial w_1}{\partial \alpha_1} \qquad \text{(IV-7)}$$

But, typically $$\frac{\partial F_{x_2}}{\partial w_1} = \frac{\partial F_{x_2}}{\partial w_2}\frac{\partial w_2}{\partial w_1} + \frac{\partial F_{x_2}}{\partial v_2}\frac{\partial v_2}{\partial w_1} + \cdots + \frac{\partial F_{x_2}}{\partial q_2}\frac{\partial q_2}{\partial w_1} + \cdots \qquad \text{(IV-7a)}$$

From (3) and (4) these derivatives can be evaluated to be

$$\frac{\partial w_2}{\partial w_1} = b_{33}\ ; \ \frac{\partial v_2}{\partial w_1} = b_{23}\ ; \ \frac{\partial P_i}{\partial w_1} = 0 \qquad \text{(IV-7b)}$$

Also $$\sin\alpha_1 = w_1/V_\infty \quad ; \quad \sin\alpha_2 = w_2/V_\infty$$

Thus $$\frac{\partial w_1}{\partial \alpha_1} = V_\infty \cos\alpha_1 \quad ; \quad \frac{\partial \alpha_2}{\partial w_2} = \frac{1}{V_\infty \cos\alpha_2} \qquad \text{(IV-7c)}$$

Note also that $$\frac{\partial F_{x_2}}{\partial w_2} = \frac{\partial F_{x_2}}{\partial \alpha_2}\frac{\partial \alpha_2}{\partial w_2} = \frac{1}{V_\infty \cos\alpha_2}\frac{\partial F_{x_2}}{\partial \alpha_2} \qquad \text{(IV-7d)}$$

Now, let $$C_{x_\alpha} \equiv \frac{\partial}{\partial \alpha}\left[\frac{F_x}{(\frac{1}{2}\rho V_\infty^2)A}\right] \qquad \text{(IV-7e)}$$

Substitution of equations (7a) through (7e) back into equation (7) yields, for a case where certain derivatives are zero (e.g. $\partial F_{y_2}/\partial w_1 = 0$),

$$C_{x_{\alpha_1}} = \lambda_1\left[\cos\delta\cos^2\epsilon\cos\sigma\, C_{x_{\alpha_2}} + \tfrac{1}{2}\left(\cos\delta\sin 2\epsilon\cos^2\sigma + \sin\delta\cos\epsilon\sin 2\sigma\right) C_{z_{\alpha_2}}\right]$$

$$- \lambda_2\left[\tfrac{1}{2}\cos\delta\sin 2\epsilon\, C_{x_{u_2}} + \left(\cos\delta\sin^2\epsilon\cos\sigma + \sin\delta\sin\epsilon\sin\sigma\right) C_{z_{u_2}}\right] \qquad \text{(IV-7f)}$$

where $$\lambda_1 = \frac{\cos\alpha_1}{\cos\alpha_2} \quad ; \quad \lambda_2 = \cos\alpha_1$$

It can be seen from this equation that for small angular misalignments this particular coefficient will experience only small changes. The actual forms of the transformation equations for any particular test item will depend on the magnitude of the misalignment, so simple algebraic forms may result in some cases.

7. Transformation Sensitivity Equations

In order to evaluate the magnitude of axis-offset effects on the various parameters of interest, sensitivity relations can be derived. These terms can then be evaluated for any specific case to determine whether the changes in parameters are significant or not. By performing this calculation prior to simulation it may be possible to reduce the computing time for some configurations.

Typical results are shown below for an intertia component and an aerodynamic coefficient. A table of equations for other parameters is presented in Appendix A.

The sensitivity equation for the inertia component, I_{xx2}, is given by (for a typical angular deviation)

$$\frac{\partial I_{xx_2}}{\partial \delta} = (I_{yy_1} - I_{xx_1}) \sin 2\delta \cos^2 \epsilon - 2 I_{xy_1} \cos 2\delta \cos^2 \epsilon - \sin 2\epsilon \, (I_{xz_1} \sin\delta - I_{yz_1} \cos\delta) \quad \text{(IV-8a)}$$

The corresponding equation for a translational deviation is given by

$$\frac{\partial I_{xx_2}}{\partial f} = m \left[2f(1 - \cos^2\delta \cos^2\epsilon) - g(\sin 2\delta \cos^2\epsilon) + h(\cos\delta \sin 2\epsilon) \right] \quad \text{(IV-8b)}$$

A similar set of sensitivity parameters for the aerodynamic coefficient, $C_{x_{\alpha_1}}$, is given by

$$\frac{\partial C_{x_{\alpha_1}}}{\partial \delta} = \lambda_1 \left[-\sin\delta \left(\cos^2\epsilon \cos\sigma \, C_{x_{\alpha_2}} + \tfrac{1}{2} \sin 2\epsilon \sin 2\sigma \, C_{z_{\alpha_2}} \right) + \tfrac{1}{2} \cos\delta \cos\epsilon \sin 2\sigma \, C_{z_{\alpha_2}} \right] + \lambda_2 \left[-\sin\delta \left(\tfrac{1}{2} \sin 2\epsilon \, C_{x_{u_2}} + \sin^2\epsilon \cos\sigma \, C_{z_{u_2}} \right) + \cos\delta \cos\epsilon \sin 2\sigma \, C_{z_{u_2}} \right] \quad \text{(IV-9a)}$$

$$\frac{\partial C_{x_{\alpha_1}}}{\partial f} = 0 \quad \text{(IV-9b)}$$

The availability of sensitivity equations such as equations (8) and (9) makes possible a determination of the relative magnitudes of the deviations prior to performing simulations.

SECTION V

RESEARCH PLAN

1. Introduction

The preceding sections of this report have described specific tasks which were performed to provide background information and to generate operational tools which would be used to perform more detailed study and development of aerodynamic models to be used in the analysis of aeroballistic test data. The intent of such a study would be to improve the analysis capabilities for conventional munitions but also to extend the envelope of applicability of ballistic range tests.

In this section a brief discussion of a research plan directed toward such analysis improvements will be given. Block diagrams identifying each phase of the project will be discussed, and, to the degree possible prior to actual initiation of the research effort, the methodology to be employed will be identified.

2. Simulation Methodology

A significant part of the aerodynamic model study will be performed with the aid of a six degree-of-freedom computer program which solves the equations of motion for a complex dynamic system. A basic program written at Litton Industries (Reference 21), the MOD6DF program, is used for this purpose. The program is structured in modular form so that the integration algorithm does not have to be altered for various applications of the program. In fact alterations of the aerodynamic models contained in the equations of motion can be readily changed without affecting the remainder of the program.

A set of operational subroutines has been prepared for use within the basic MOD6DF structure. These subroutines contain the equations of motion and the current aerodynamic models which are being used in ballistic data analysis at Eglin AFB. The equations as they appear in the program listing are shown in Appendix B. These equations were developed by General Electric and are shown in detail in Reference 13. The current aerodynamic models are in polynomial form. However, the program structure is such that any analytical model could be used.

The standard output package of the MOD6DF program has been modified to be compatibile with data acquired in the ballistic range. Output can be commanded to occur at the downrange positions which coincide with data stations in the range. This simplifies the procedures for comparing simulated and free flight data. Of course, time increment outputs are also available.

An additional subroutine containing the transformation equations shown in Section IV of this report has also been included in the program. This subroutine, listed in Appendix B, permits direct accounting for mass offsets in the simulation.

3. Estimation Methodology

The parameter estimation programs at Eglin AFB will be used in the data analysis phase of the study. These programs are described in Reference 13. The maximum likelihood method of estimation is used in the programs. Thus, the aerodynamic models and the sensitivity parameters which involve the models analytically are "hard wired" into the program. Spontaneous changes in the models incorporated into the extraction programs are not possible. This implies that the principal use of these programs will be in performing studies with the existing models to determine extraction-order sensitivites and model bias. If significant model forms which have been proven by simulation studies do become available, the extraction programs will be altered to accommodate them.

Several methods of parameter estimation were discussed in Section II of this report. The model structure determination (MSD) method appeared to have considerable promise with regard to the present research objectives. However, no work is planned in this area. Rather, the effort will be directed toward improvement of the existing MLM program.

4. Interactive Methodology

It is intended that most of the analytical studies be performed in the interactive mode at the graphics terminal. The Tektronix 4014 unit is a large screen terminal with extensive graphics capability. A program such as the MOD6DF simulation program can be run from the terminal with graphics output. A software package has been prepared which will permit a frame of data to be stored and then recalled in any scale and to any part of the screen. This software allows frames to be multiply overlaid so that simulation runs with different model forms can be instantly compared with each other and with test data. This feature will permit rapid identification of specific characteristics of the various models. Of course, the numerical information which is displayed graphically on the screen is also readily available for detailed study.

5. Synthesis Methodology

A block diagram of the interactive computational system is shown in Figure . Note that the parameter estimation programs are actually installed and operational on the Eglin AFB CDC 6600 computer system. These programs will be run by remote telephone connection on the graphics terminal at the research site. This procedure eliminates costly installation and running expenses at the research site.

All of the simulation work is done on site at the graphics terminal. Since this operation is performed within Department facilities, it is not time restricted.

6. Scope of Research

The overall purpose of the continuing research program is to provide improved aerodynamic model forms for use in ballistic range data analysis. Three major tasks can be identified in this regard. These are:

(1) Determination of the effects of coefficient extraction order on the accuracy of results, (2) Aerodynamic model improvements through the use of alternative model forms, and (3) Correlation of model form selection with fundamental aerodynamic flow field phenomena. Of course, these tasks are interdependent and cannot be separated on a phenomenological basis, but the methodology to be employed in each task can and will be discussed separately. It should also be pointed out that the effort will be directed toward studies of tail-finned configurations only.

Apparently, the quality of the parameter estimations performed by the AFATL maximum likelihood program is affected by the order in which extractions are made. This is probably related directly to the analytical model which is chosen to represent the physical system in the estimation procedure. In the present research no effort will be made to derive uniqueness theorems (or lack thereof) for the estimation procedure. Rather the method will be applied repetitively for various models and configurations in an attempt to establish model-related trends. In this study the Aeroballistic Research Facility Data Analysis System (ARFDAS) developed by General Electric Company for AFATL will be used. The existing aerodynamic models will be used in this study initially since a considerable supply of data is available for comparison with the present analysis. The technique to be employed is straight-forward: perform the parameter estimation procedure systematically on different sets of coefficients in the model, compare the results of these studies with each other and with existing data, and formulate trends in the results. Of course, systematic manipulation in such a manner for all combinations of coefficients would be prohibitively expensive, so coefficients which have historically been sensitive to such manipulations will be isolated for the study. Certain axial force coefficients and all rolling motion coefficients are prime candidates for this treatment.

The benefits to be gained from this type of analysis are potentially great, but there are several uncertainties in the analysis. Principal among these is the degree to which the effect of extraction order is dependent on the model form. The studies will be done only with existing models, so results achieved may be inconclusive in the general sense. The statistical nature of the estimation procedure will make it very difficult to determine why a particular trend occurs. In addition, it may turn out that order changes may produce opposite effects in different parameters. If such occurrences cannot be resolved through correlation with the actual aerodynamics, little of value will come out of the study. To date, no comprehensive investigation has been performed in this area, so the direct method of analysis is a logical choice. In any case, the expected result of the study is an indication of the type and degree of interaction which exists between the order and the accuracy of the estimated coefficients.

The remaining two tasks associated with the project are very closely related. Certainly, the aerodynamic flow field produced around the flying finned body directly determines the aerodynamic forces and moments on the body which dictate its motion. The analytical model of

these aerodynamic forces and moments which is used in the data analysis algorithm should also be influenced by the physics of the flow field. Reference 13 contains the analytical models which are currently in use at AFATL. For many cases, these models yield splendid results. However, there is still some uncertainty in the models for some cases. For example, in one recent test the motion of the test item was very well predicted except for the roll orientation. In another, the axial force estimates were apparently in error. Thus, some model improvements are necessary, but more importantly, a systematic method for selecting a model form is sought.

Again, a direct method of analysis is suggested. The utility of the method will be enhanced by the use of computer graphics techniques which permit rapid comparisons of a large number of model forms and flow field effects. A typical model analysis procedure can be outlined as follows:

(1) Experimental data from a test of a finned body are stored.
(2) The results of a parameter estimation run for that case using the existing aerodynamic models are determined and stored.
(3) A 6DOF simulation of the test item motion using the extracted coefficients is performed and stored to be used as an analytical baseline.
(4) Simulations using variations of the baseline analytical model are performed, and the graphical results are stored by frame.
(5) The extracted simulation, the experimental data and the results using new aerodynamic models are then manipulated interactively at the graphics terminal to evaluate the influence of the model changes on the simulated results. Frequencies, amplitudes and phase information are compared for the various cases to determine model change effects.

The present research in model form will not be confined to seeking higher order effects in the conventional expansion of aerodynamic forces and moments. Some forms of aerodynamic data may suggest unexpected types of non-linearities which cannot be treated in the usual manner. An example of this type which has already been observed is a drag non-linearity which is not symmetrical and, thus, cannot be modeled with an even-power expansion. Other new forms may also be dictated by the unique aerodynamic regimes in which some of the new munitions operate. High angle of incidence is common in these casses, and the large separated regions and strong vortices produce unusual behavior. Because the simulation programs to be used in the study are modular, the new aerodynamic model forms which may be dictated by these flow fields can be easily incorporated into the programs. Even for radical model forms, the procedure to be followed will be as outlined above. It should be understood, however, that these new models will not be incorporated into the parameter estimation programs until the selected forms have been shown to be superior for several different configurations.

Correlation of flow field information with aerodynamic model form will be accomplished by analysis of wind tunnel data, flow field pictures and ballistic test data. The emphasis here will be on attempting to

detect specific features of the flow field which manifest themselves in certain aspects of the model form. As an example, an induced rolling moment may be attributable to body-fin interference which could be described in the aerodynamic model in terms of certain aerodynamic or geometric parameters. The basic procedure here will be to seek trends in behavior and to relate those trends to the model form.

The proposed method for dealing with the aerodynamic model forms is direct. Such an approach may be expected to discover major deviations from conventional model forms. It may not, however, provide detailed information on subtle flow field effects on the models. In such cases where obvious discrepancies occur, additional perturbations of the evaluation procedure may be necessary. Although the nature of these possible deviations cannot be predicted now, it is felt that the basic tools of analysis are sufficiently flexible to allow for them in some manner. Systematic variation of model form within the simulation programs would be possible, and the effects of these variations on specific parameters could be investigated using the frame overlay capability of the graphics terminal.

SECTION VI

CONCLUSIONS

In the foregoing report a research plan has been described, and the basic computational tools required to implement the plan have been developed. The study has indicated that model improvements are necessary if the envelope of applicability of the ballistic range as a test facility is to be expanded to include airplane-like test items. The approach to improving these models has been outlined here. The present study has been concerned with exploring the possibilities of research in this area. An effort to perform model development has not been made.

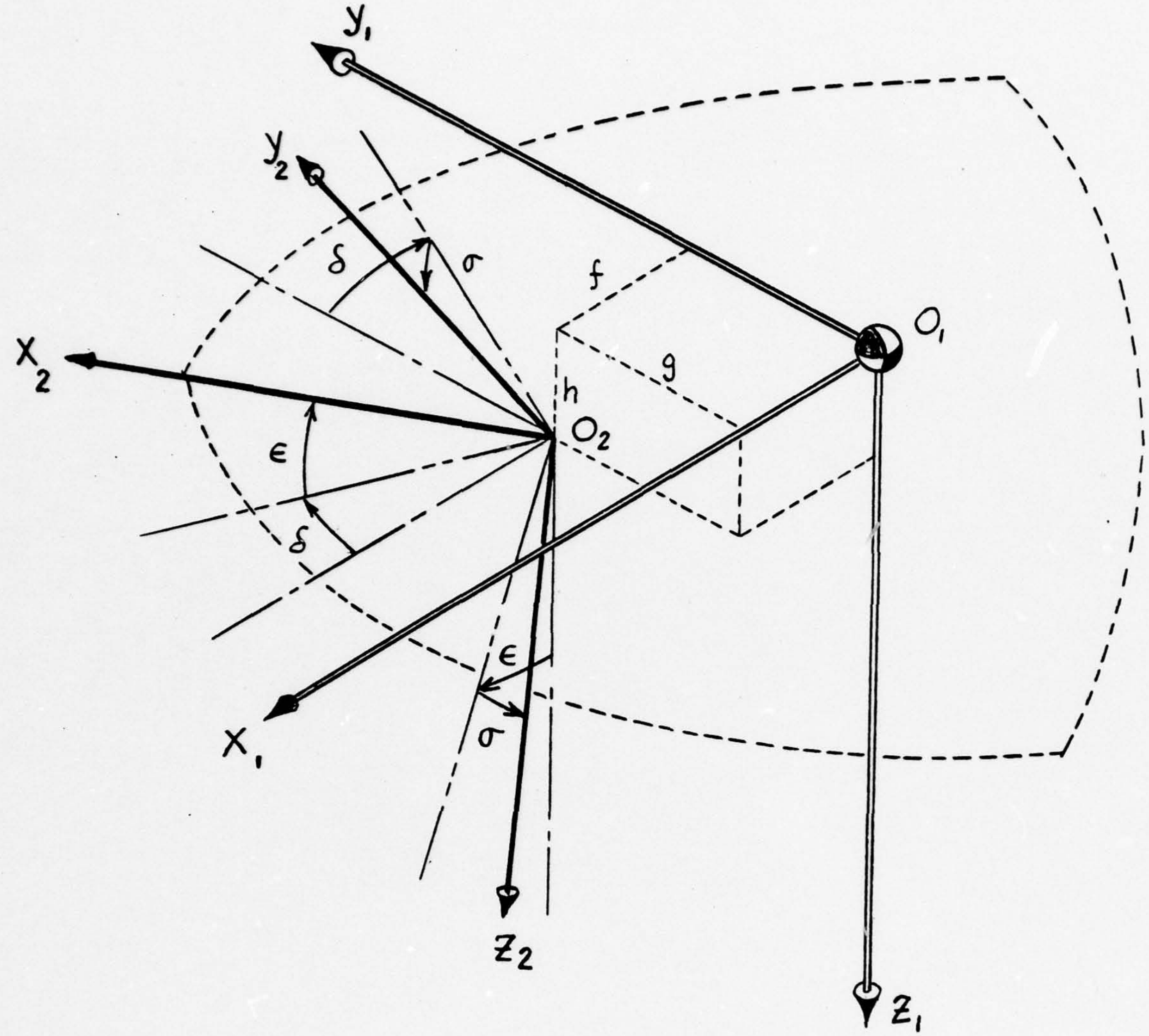

FIGURE 1. TRANSFORMATION NOMENCLATURE

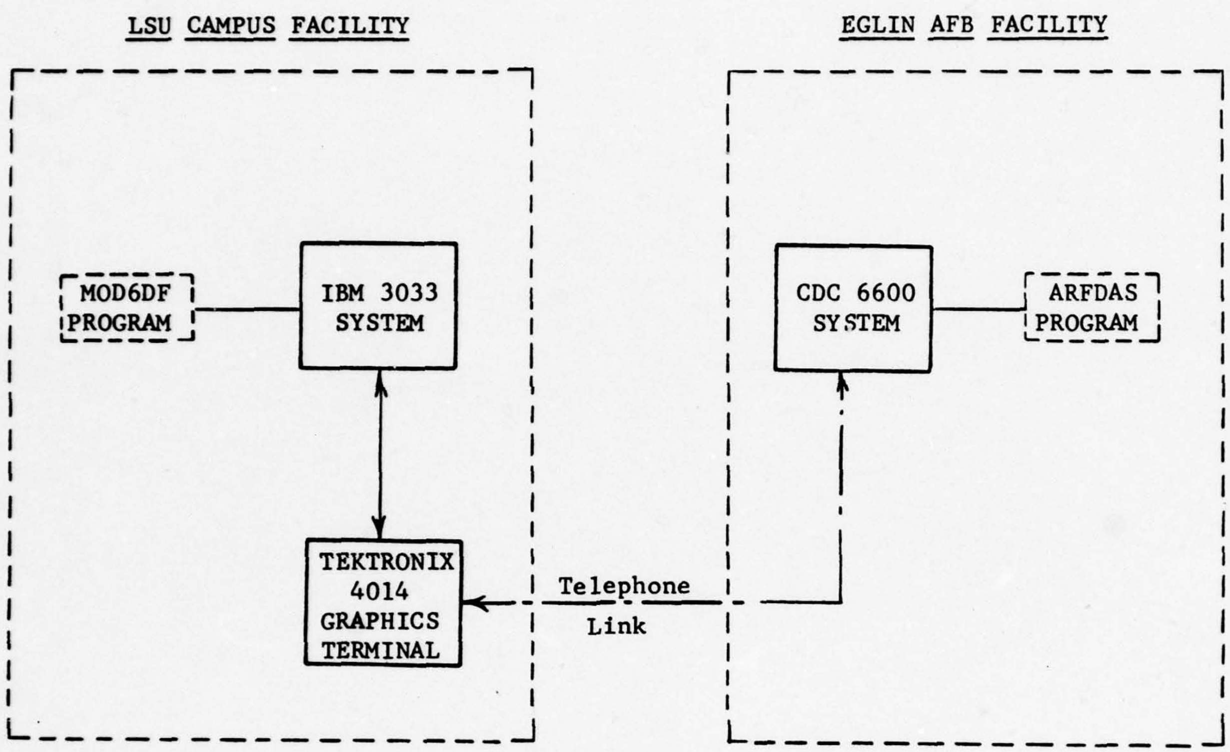

FIGURE 2. COMPUTATIONAL FACILITIES

APPENDIX A

TRANSFORMATION EQUATIONS

1. Matrix elements, b_{ij}

$$[B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \qquad (A\text{-}1)$$

where

$$\begin{aligned}
b_{11} &= \cos\delta \cos\epsilon \\
b_{12} &= \sin\delta \cos\epsilon \\
b_{13} &= -\sin\epsilon \\
b_{21} &= \cos\delta \sin\epsilon \sin\sigma - \sin\delta \cos\sigma \\
b_{22} &= \sin\delta \sin\epsilon \sin\sigma + \cos\delta \cos\sigma \\
b_{23} &= \cos\epsilon \sin\sigma \\
b_{31} &= \cos\delta \sin\epsilon \cos\sigma + \sin\delta \sin\sigma \\
b_{32} &= \sin\delta \sin\epsilon \cos\sigma - \cos\delta \sin\sigma \\
b_{33} &= \cos\epsilon \cos\sigma
\end{aligned}$$

Equation (A-1) is the matrix (or its transpose) which appears in equations (1) and (3) through (7) of Section IV.

2. Matrix elements, C_{ij}

The C matrix appears in the inertia transformation which is given by equation (2) as

$$\begin{Bmatrix} I_{xx_2} \\ I_{xy_2} \\ I_{xz_2} \\ I_{yy_2} \\ I_{yz_2} \\ I_{zz_2} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} I_{xx_1} \\ I_{xy_1} \\ I_{xz_1} \\ I_{yy_1} \\ I_{yz_1} \\ I_{zz_1} \end{Bmatrix} + \begin{Bmatrix} G_{xx} \\ G_{xy} \\ G_{xz} \\ G_{yy} \\ G_{yz} \\ G_{zz} \end{Bmatrix} \qquad (A\text{-}2)$$

where

$$C_{11} = \cos^2\delta \cos^2\epsilon$$
$$C_{12} = -\sin 2\delta \cos^2\epsilon$$
$$C_{13} = \cos\delta \sin 2\epsilon$$
$$C_{14} = \sin^2\delta \cos^2\epsilon$$
$$C_{15} = \sin\delta \sin 2\epsilon$$
$$C_{16} = \sin^2\epsilon$$

$$C_{21} = \cos\delta \cos\epsilon (\cos\delta \sin\epsilon \sin\sigma - \sin\delta \cos\sigma)$$
$$C_{22} = \cos 2\delta \cos\epsilon \cos\sigma + \tfrac{1}{2} \sin 2\delta \sin 2\epsilon \sin\sigma$$
$$C_{23} = \cos\delta \cos 2\epsilon \sin\sigma + \sin\delta \sin\epsilon \cos\sigma$$
$$C_{24} = -\tfrac{1}{2}[(1+\cos^2\delta)\sin 2\epsilon \sin\sigma - \sin 2\delta \cos\epsilon \cos\sigma]$$
$$C_{25} = \sin\delta \cos 2\epsilon \sin\sigma - \cos\delta \sin\epsilon \cos\sigma$$
$$C_{26} = \sin\epsilon \cos\epsilon \sin\sigma$$

$$C_{41} = \sin^2\delta \cos^2\sigma + \cos^2\delta \sin^2\epsilon \sin^2\sigma - \tfrac{1}{2} \sin 2\delta \sin\epsilon \sin 2\sigma$$
$$C_{42} = \sin 2\delta (\cos^2\sigma - \sin^2\epsilon \sin^2\sigma) - \cos 2\delta \sin\epsilon \sin 2\sigma$$
$$C_{43} = \sin\delta \cos\epsilon \sin 2\sigma - \cos\delta \sin 2\epsilon \sin^2\sigma$$
$$C_{44} = \cos^2\delta \cos^2\sigma + \sin^2\delta \sin^2\epsilon \sin^2\sigma + \tfrac{1}{2} \sin 2\delta \sin\epsilon \sin 2\sigma$$
$$C_{45} = \cos\delta \cos\epsilon \sin 2\sigma + \sin\delta \sin 2\epsilon \sin^2\sigma$$
$$C_{46} = \cos^2\epsilon \sin^2\sigma$$

$$C_{31} = -\tfrac{1}{2}(\cos^2\delta \sin 2\epsilon \cos\sigma + \sin 2\delta \cos\epsilon \sin\sigma)$$
$$C_{32} = \tfrac{1}{2}(\sin 2\delta \sin 2\epsilon \cos\sigma) - \cos 2\delta \cos\epsilon \sin\sigma$$
$$C_{33} = \cos\delta \cos 2\epsilon \cos\sigma - \sin\delta \sin\epsilon \sin\sigma$$
$$C_{34} = \sin\delta \cos\epsilon (\cos\delta \sin\sigma - \sin\delta \sin\epsilon \cos\sigma)$$
$$C_{35} = \sin\delta \cos 2\epsilon \cos\sigma + \cos\delta \sin\epsilon \sin\sigma$$
$$C_{36} = \sin\epsilon \cos\epsilon \cos\sigma$$

$$C_{51} = \tfrac{1}{2}(\sin 2\delta \sin\epsilon \cos 2\sigma - \cos^2\delta \sin^2\epsilon \sin 2\sigma)$$
$$C_{52} = \tfrac{1}{2} \sin 2\delta (\sin^2\epsilon + 1) \sin 2\sigma + \sin\epsilon (\cos 2\delta \cos^2\sigma + \cos^2\delta \cos 2\sigma)$$
$$C_{53} = \tfrac{1}{2} \cos\delta \sin 2\epsilon \sin 2\sigma - \sin\delta \cos\epsilon \cos 2\sigma$$
$$C_{54} = \tfrac{1}{2}[\sin 2\sigma (1 - \sin^2\delta \sin^2\epsilon) - \sin 2\delta \sin\epsilon \cos 2\sigma]$$
$$C_{55} = \tfrac{1}{2} \sin\delta \sin 2\epsilon \sin 2\sigma + \cos\delta \cos\epsilon \cos 2\sigma$$
$$C_{56} = -\tfrac{1}{2} \cos^2\epsilon \sin 2\sigma$$

$$C_{61} = \cos^2\delta \sin^2\epsilon \cos^2\sigma + \sin^2\delta \sin^2\sigma + \tfrac{1}{2} \sin 2\delta \sin\epsilon \sin 2\sigma$$
$$C_{62} = \sin 2\delta [1 - \cos^2\sigma (\sin^2\epsilon + 1)] + \cos 2\delta \sin\epsilon \sin 2\sigma$$
$$C_{63} = -\sin\delta \cos\epsilon \sin 2\sigma - \cos\delta \sin 2\epsilon \cos^2\sigma$$
$$C_{64} = \sin^2\delta \sin^2\epsilon \cos^2\sigma + \cos^2\delta \sin^2\sigma - \tfrac{1}{2} \sin 2\delta \sin\epsilon \sin 2\sigma$$
$$C_{65} = \cos\delta \cos\epsilon \sin 2\sigma - \sin\delta \sin 2\epsilon \cos^2\sigma$$
$$C_{66} = \cos^2\epsilon \cos^2\sigma$$

and also,

$$G_{xx} = m\Big[(1-\cos^2\delta\cos^2\epsilon)f^2 + (1-\sin^2\delta\cos^2\epsilon)g^2 + \cos^2\epsilon\, h^2$$
$$-fg(\sin 2\delta\cos^2\epsilon) + gh(\sin\delta\sin 2\epsilon) + fh(\cos\delta\sin 2\epsilon)\Big]$$

$$G_{xy} = m\Big[(\tfrac{1}{2}\cos^2\delta\sin 2\epsilon\sin\sigma - \tfrac{1}{2}\sin 2\delta\cos\epsilon\cos\sigma)f^2 + (\tfrac{1}{2}\sin^2\delta\sin 2\epsilon\sin\sigma$$
$$+\tfrac{1}{2}\sin 2\delta\cos\epsilon\cos\sigma)g^2 - (\tfrac{1}{2}\sin 2\epsilon\sin\sigma)h^2 + (\cos 2\delta\cos\epsilon\cos\sigma$$
$$+\tfrac{1}{2}\sin 2\delta\sin 2\epsilon\sin\sigma)fg + (\cos\delta\cos 2\epsilon\sin\sigma + \sin\delta\sin\epsilon\cos\sigma)fh$$
$$+(\sin\delta\cos 2\epsilon\sin\sigma - \cos\delta\sin\epsilon\cos\sigma)gh\Big]$$

$$G_{xz} = m\Big[(\tfrac{1}{2}\cos^2\delta\sin 2\epsilon\cos\sigma + \tfrac{1}{2}\sin 2\delta\cos\epsilon\sin\sigma)f^2 + (\tfrac{1}{2}\sin^2\delta\sin 2\epsilon\cos\sigma$$
$$-\tfrac{1}{2}\sin 2\delta\cos\epsilon\sin\sigma)g^2 - (\tfrac{1}{2}\sin 2\epsilon\cos\sigma)h^2 + (\tfrac{1}{2}\sin 2\delta\sin 2\epsilon\cos\sigma$$
$$-\cos 2\delta\cos\epsilon\sin\sigma)fg + (\sin\delta\cos 2\epsilon\cos\sigma + \cos\delta\sin\epsilon\sin\sigma)gh$$
$$+(\cos\delta\cos 2\epsilon\cos\sigma - \sin\delta\sin\epsilon\sin\sigma)fh\Big]$$

$$G_{yy} = m\Big\{\big[\cos^2\delta(1-\sin^2\epsilon\sin^2\sigma) + \sin^2\delta\sin^2\sigma + \tfrac{1}{2}\sin 2\delta\sin\epsilon\sin 2\sigma\big]f^2$$
$$+\big[\sin^2\delta(1-\sin^2\epsilon\sin^2\sigma) + \cos^2\delta\sin^2\sigma - \tfrac{1}{2}\sin 2\delta\sin\epsilon\sin 2\sigma\big]g^2$$
$$+(\sin^2\epsilon + \cos^2\epsilon\cos^2\sigma)h^2 + \big[\sin 2\delta(\cos^2\sigma - \sin^2\epsilon\sin^2\sigma) - \cos 2\delta\sin\epsilon\sin 2\sigma\big]fg$$
$$+(\sin\delta\cos\epsilon\sin 2\sigma - \cos\delta\sin 2\epsilon\sin^2\sigma)fh + (\cos\delta\cos\epsilon\sin 2\sigma$$
$$+\sin\delta\sin 2\epsilon\sin^2\sigma)gh\Big\}$$

$$G_{yz} = m\{[\tfrac{1}{2}\sin 2\sigma(\cos^2\delta\sin^2\epsilon + \sin^2\delta) - \tfrac{1}{2}\sin 2\delta\sin\epsilon\cos 2\sigma]f^2$$

$$+ [\tfrac{1}{2}\sin 2\sigma(\sin^2\delta\sin^2\epsilon - \cos^2\delta) + \tfrac{1}{2}\sin 2\delta\sin\epsilon\cos 2\sigma]g^2$$

$$+ (\tfrac{1}{2}\cos^2\epsilon\sin 2\sigma)h^2 + (\tfrac{1}{2}\sin\delta\sin 2\epsilon\sin 2\sigma + \cos\delta\cos\epsilon\cos 2\sigma)gh$$

$$+ [\tfrac{1}{2}\sin 2\delta(\sin^2\epsilon + 1)\sin 2\sigma + \sin\epsilon(\cos 2\delta\cos^2\sigma + \cos^2\delta\cos 2\sigma)]fg$$

$$+ (\tfrac{1}{2}\cos\delta\sin 2\epsilon\sin 2\sigma - \sin\delta\cos\epsilon\cos 2\sigma)fh\}$$

$$G_{zz} = m\{[\cos^2\delta(1 - \sin^2\epsilon\cos^2\sigma) + \sin^2\delta\cos^2\sigma - \tfrac{1}{2}\sin 2\delta\sin\epsilon\sin 2\sigma]f^2$$

$$+ [\sin^2\delta(1 - \sin^2\epsilon\cos^2\sigma) + \cos^2\delta\cos^2\sigma + \tfrac{1}{2}\sin 2\delta\sin\epsilon\sin 2\sigma]g^2$$

$$+ (\sin^2\epsilon + \cos^2\epsilon\sin^2\sigma)h^2 + (\cos\delta\cos\epsilon\sin 2\sigma - \sin\delta\sin 2\epsilon\cos^2\sigma)gh$$

$$+ [\sin 2\delta(1 - \cos^2\sigma[\sin^2\epsilon + 1]) + \cos 2\delta\sin\epsilon\sin 2\sigma]fg$$

$$- (\sin\delta\cos\epsilon\sin 2\sigma + \cos\delta\sin 2\epsilon\cos^2\sigma)fh\}$$

3. <u>Aerodynamic Coefficients</u>

$$C_{X_{\alpha_1}} = \lambda_1[\cos\delta\cos^2\epsilon\cos\sigma\, C_{X_{\alpha_2}} + \tfrac{1}{2}(\cos\delta\sin 2\epsilon\cos^2\sigma$$

$$+ \sin\delta\cos\epsilon\sin 2\sigma)C_{Z_{\alpha_2}}] + \lambda_2[-\tfrac{1}{2}\cos\delta\sin 2\epsilon\, C_{X_{u_2}}$$

$$-(\cos\delta\sin^2\epsilon\cos\sigma + \sin\delta\sin\epsilon\sin\sigma)C_{Z_{u_2}}]$$

$$C_{y_{\beta_1}} = \frac{\lambda_1 \lambda_4}{\lambda_2 \lambda_3} \Big\{ C_{x_{\alpha_2}} \Big[\tfrac{1}{4} (\sin 2\delta \sin 2\sigma (\sin^2 \epsilon + 1)) - \sin\epsilon \, (\sin^2\delta \cos^2\sigma$$

$$+ \cos^2\delta \sin^2\sigma) \Big] + C_{z_{\alpha_2}} \Big[\tfrac{1}{4} \sin\delta \sin 2\epsilon \sin 2\sigma - \cos\delta \cos\epsilon$$

$$\sin^2\sigma \Big] \Big\} + \lambda_4 \, C_{y_{\beta_2}} (\sin\delta \sin\epsilon \sin\sigma + \cos\delta \cos\sigma)^2$$

$$C_{z_{\alpha_1}} = \lambda_1 \Big[C_{x_{\alpha_2}} (\cos\delta \cos^2\sigma \sin\epsilon \cos\epsilon + \tfrac{1}{2} \sin\delta \cos\epsilon \sin 2\sigma)$$

$$+ C_{z_{\alpha_2}} (\cos^2\epsilon \cos^2\sigma) \Big]$$

$$C_{l_{p_1}} = \cos^2\delta \cos^2\epsilon \, C_{l_{p_2}} + (\tfrac{1}{2} \cos^2\delta \sin 2\epsilon \cos\sigma + \tfrac{1}{2} \sin 2\delta \cos\epsilon$$

$$\sin\sigma) C_{n_{p_2}} + \frac{2b\lambda_3}{V_\infty^2} \Big\{ C_{l_{\beta_2}} \big[\cos\delta \cos^2\epsilon \sin\sigma \, g$$

$$- \tfrac{1}{2} \sin 2\delta \cos^2\epsilon \, h \big] + C_{n_{\beta_2}} \big[\tfrac{1}{4} \cos\delta \sin 2\epsilon \sin 2\sigma \, g$$

$$- (\sin\delta \sin\epsilon \sin\sigma + \cos\delta \cos\sigma)(\cos\delta \sin\epsilon \cos\sigma + \sin\delta \sin\sigma) h \big] \Big\}$$

$$C_{m_{\dot\alpha_1}} = C_{m_{\dot\alpha_2}} \lambda_1 (\cos\delta \cos\epsilon \cos^2\sigma + \tfrac{1}{4} \sin\delta \sin 2\epsilon \sin 2\sigma)$$

$$+ C_{x_{\dot\alpha_2}} \frac{\lambda_2}{c} \big[h (\cos\delta \cos^2\epsilon \cos\sigma) + f (\tfrac{1}{2} \sin 2\epsilon \cos\sigma) \big]$$

$$+ C_{z_{\dot\alpha_2}} \frac{\lambda_2}{c} \big[\tfrac{h}{2} (\cos\delta \sin 2\epsilon \cos^2\sigma + \sin\delta \cos\epsilon \sin 2\sigma)$$

$$- f \cos^2\epsilon \cos^2\sigma \big]$$

$$
\begin{aligned}
C_{n_{\beta_1}} = {} & C_{n_{\beta_2}} \lambda_3 \left(\cos\delta \cos\epsilon \cos^2\sigma + \tfrac{1}{4} \sin\delta \sin 2\epsilon \sin 2\sigma \right) \\
& - C_{\ell_{\beta_2}} \lambda_4 \left(\cos\delta \sin\epsilon \cos\sigma + \sin\delta \sin^2\epsilon \sin\sigma \right) \\
& + C_{y_{\beta_2}} \left(\frac{\lambda_3}{b} \right) \Big[f \left(\sin\delta \sin\epsilon \sin\sigma + \cos\delta \cos\sigma \right)^2 \\
& - g \left(\tfrac{1}{2} \sin 2\delta \left[\sin^2\epsilon \sin^2\sigma - \cos^2\sigma \right] - \tfrac{1}{2} \cos 2\delta \sin\epsilon \sin 2\sigma \right) \Big]
\end{aligned}
$$

APPROXIMATE SENSITIVITIES FOR INERTIAS

	$\partial/\partial\delta$	$\partial/\partial\epsilon$	$\partial/\partial\sigma$	$\partial/\partial f$	$\partial/\partial g$	$\partial/\partial h$
I_{xx_2}	$(I_{yy_1}-I_{xx_1})2\delta-2I_{xy_1}$	$(I_{zz_1}-I_{xx_1})2\epsilon+2I_{xz_1}$	0	0	$2mg$	$2mh$
I_{yy_2}	$(I_{xx_1}-I_{yy_1})2\delta+2I_{xy_1}$	0	$(I_{zz_1}-I_{yy_1})2\sigma+2I_{yz_1}$	$2mf$	0	$2mh$
I_{zz_2}	0	$(I_{xx_1}-I_{zz_1})2\epsilon-2I_{xz_1}$	$(I_{yy_1}-I_{zz_1})2\sigma+2I_{yz_1}$	$2mf$	$2mg$	0
I_{xy_2}	$(I_{yy_1}-I_{xx_1})-4\delta I_{xy_1}$	$-I_{xy_1}\epsilon-I_{yz_1}$	$-I_{xy_1}\sigma+I_{xz_1}$	$2mg$	$2mf$	0
I_{yz_2}	$-I_{xz_1}\delta+I_{yz_1}$	$2I_{xy_1}-I_{yz_1}\epsilon$	$(I_{yy_1}-I_{zz_1})-4\sigma I_{yz_1}$	0	$2mh$	$2mg$
I_{xz_2}	$-I_{xz_1}\delta+I_{yz_1}$	$(I_{zz_1}-I_{xx_1})2\epsilon-4\epsilon I_{xz_1}$	$-I_{xz_1}\sigma+I_{xy_1}$	$2mh$	0	$2mf$

APPENDIX B

B.1 TRANSLATIONAL DYNAMICS MODULE

```
      SUBROUTINE D1
C     TRANSLATIONAL DYNAMICS MODULE D1B - BODY AXES
      COMMON C(3415)
      REAL MASS
      EQUIVALENCE (C(1217),MASS)
      EQUIVALENCE     (C(0932)  , T)
      EQUIVALENCE   (C(0073)  , TXBA     )
      EQUIVALENCE   (C(0074)  , TYBA     )
      EQUIVALENCE   (C(0075)  , TZBA     )
      EQUIVALENCE   (C(0086)  , WE       )
      EQUIVALENCE   (C(0128)  , FXBA     )
      EQUIVALENCE   (C(0129)  , FYBA     )
      EQUIVALENCE   (C(0130)  , FZBA     )
      EQUIVALENCE (C(0224),A11   )
      EQUIVALENCE (C(0228),A12   )
      EQUIVALENCE (C(0232),A13   )
      EQUIVALENCE (C(0236),A21   )
      EQUIVALENCE (C(0240),A22   )
      EQUIVALENCE (C(0244),A23   )
      EQUIVALENCE (C(0248),A31   )
      EQUIVALENCE (C(0252),A32   )
      EQUIVALENCE (C(0256),A33   )
      EQUIVALENCE   (C(0282)  , AG       )
      EQUIVALENCE   (C(0283)  , VXDTP    )
      EQUIVALENCE   (C(0286)  , VXTP     )
      EQUIVALENCE   (C(0287)  , XDTP     )
      EQUIVALENCE   (C(0290)  , XTP      )
      EQUIVALENCE   (C(0291)  , VYDTP    )
      EQUIVALENCE   (C(0294)  , VYTP     )
      EQUIVALENCE   (C(0295)  , YDTP     )
      EQUIVALENCE   (C(0298)  , YTP      )
      EQUIVALENCE   (C(0299)  , VZDTP    )
      EQUIVALENCE   (C(0302)  , VZTP     )
      EQUIVALENCE   (C(0303)  , ZDTP     )
      EQUIVALENCE   (C(0411)  , GCX      )
      EQUIVALENCE   (C(0306)  , ZTP      )
```

```
      EQUIVALENCE  (C(0412) , GCY    )
      EQUIVALENCE  (C(0413) , GCZ    )
      EQUIVALENCE  (C(0710) , TAXBA  )
      EQUIVALENCE  (C(0711) , TAYBA  )
      EQUIVALENCE  (C(0712) , TAZBA  )
      EQUIVALENCE  (C(0713) , TAX)
      EQUIVALENCE (C(0714)  , TAY)
      EQUIVALENCE  (C(0715)  , TAZ)
      EQUIVALENCE (C(0509),UBA)
      EQUIVALENCE (C(0510),VBA)
      EQUIVALENCE (C(0511),WBA)
      EQUIVALENCE (C(0212),PBA)
      EQUIVALENCE (C(0216),QBA)
      EQUIVALENCE (C(0220),RBA)
      EQUIVALENCE (C(1202),XLIM)
 2000 FORMAT(1X,'ENTER D1')
      AGWE=1.0/MASS
C     TOTAL ACCELERATION DUE TO AERODYNAMIC FORCES AND THRUST FORCES
      TAXBA = AGWE*FXBA - WBA*QBA + VBA*RBA
      TAYBA = AGWE*FYBA - UBA*RBA + WBA*PBA
      TAZBA = AGWE*FZBA - VBA*PBA + UBA*QBA
C     RESOLVE FROM BODY AXES TO TANGENT PLANE
      TAX = A11*TAXBA + A21*TAYBA + A31*TAZBA
      TAY = A12*TAXBA + A22*TAYBA + A32*TAZBA
      TAZ = A13*TAXBA + A23*TAYBA + A33*TAZBA
C     INTEGRATE ACCELERATIONS DUE TO AERODYNAMICS, THRUST, GRAVITY,
C     AND CORIOLIS
      VXDTP = TAX + GCX
      VYDTP = TAY + GCY
      VZDTP = TAZ + GCZ
C     INTEGRATE VELOCITY
      XDTP = VXTP
      YDTP = VYTP
      ZDTP = VZTP
      IF(ABS(XTP-XLIM).LE.0.5)LCONV = 2
      IF((XLIM-XTP).LT.0.)LCONV = 1
```

```
RETURN
END
```

B.2 ROTATIONAL DYNAMICS MODULE

```
SUBROUTINE D2
COMMON C(3415)
EQUIVALENCE   (C(0131)  ,OLBA   )
EQUIVALENCE   (C(0132)  ,OMBA   )
EQUIVALENCE   (C(0133)  ,ONBA   )
EQUIVALENCE   (C(0150)  ,OLT    )
EQUIVALENCE   (C(0151)  ,OMT    )
EQUIVALENCE   (C(0152)  ,ONT    )
EQUIVALENCE   (C(0201)  ,A      )
EQUIVALENCE   (C(0202)  ,B      )
EQUIVALENCE   (C(0203)  ,G      )
EQUIVALENCE   (C(0204)  ,E      )
EQUIVALENCE   (C(0209)  ,PDBA   )
EQUIVALENCE   (C(0212)  ,PBA    )
EQUIVALENCE   (C(0213)  ,QDBA   )
EQUIVALENCE   (C(0216)  ,QBA    )
EQUIVALENCE   (C(0217)  ,RDBA   )
EQUIVALENCE   (C(0220)  ,RBA    )
EQUIVALENCE   (C(0221)  ,AD11   )
EQUIVALENCE   (C(0224)  ,A11    )
EQUIVALENCE   (C(0225)  ,AD12   )
EQUIVALENCE   (C(0228)  ,A12    )
EQUIVALENCE   (C(0229)  ,AD13   )
EQUIVALENCE   (C(0232)  ,A13    )
EQUIVALENCE   (C(0233)  ,AD21   )
EQUIVALENCE   (C(0236)  ,A21    )
EQUIVALENCE   (C(0237)  ,AD22   )
EQUIVALENCE   (C(0240)  ,A22    )
EQUIVALENCE   (C(0241)  ,AD23   )
EQUIVALENCE   (C(0244)  ,A23    )
EQUIVALENCE   (C(0245)  ,AD31   )
EQUIVALENCE   (C(0248)  ,A31    )
EQUIVALENCE   (C(0249)  ,AD32   )
EQUIVALENCE   (C(0252)  ,A32    )
EQUIVALENCE   (C(0253)  ,AD33   )
EQUIVALENCE   (C(0256)  ,A33    )
```

```
2000 FORMAT(1X,'ENTER D2')
     TLBA = OLBA
     TMBA = OMBA
     TNBA = ONBA
     FIP = A*G -E*E
     PDBA = (OLBA*G + ONBA*E - QBA*(E*PBA*(B-A-G) - RBA*G*(B-G-(E*E)/G)
    1))/FIP
     QDBA = (OMBA + RBA*PBA*(G-A) + E*(RBA*RBA - PBA*PBA))/B
     RDBA = (OLBA*E + ONBA*A + QBA*(E*RBA*(B-A-G) + PBA*A*(A-B-(E*E)/A)
    1))/FIP
     AD11=A21*RBA-A31*QBA
     AD12=A22*RBA-A32*QBA
     AD13=A23*RBA-A33*QBA
     AD21=A31*PBA-A11*RBA
     AD22=A32*PBA-A12*RBA
     AD23=A33*PBA-A13*RBA
     AD31=A11*QBA-A21*PBA
     AD32=A12*QBA-A22*PBA
     AD33=A13*QBA-A23*PBA
     RETURN
     END
```

B.3 TRANSFORMATION MODULE

```
      DIMENSION C(6,6),X1(6),X2(6),G(6),ROUT(99),POUT(99),TOUT(99),
     &XX(99),XY(99),XZ(99),YY(99),YZ(99),ZZ(99)
      DIMENSION BT(6,6),XL(99),XM(99),XN(99),PVARY(99)
      II=00
      READ(5,999)X1
    5 READ(5,100)DELTA,EPS,SIGMA,X,Y,Z,IGO
      READ (5,104)IVARY,XMIN,XMAX,XINC
  104 FORMAT(I10,3F10.0)
      GO TO (6,7,8,9,10,11),IVARY
    6 DELTA=XMIN/57.2958
      GO TO 12
    7 EPS=XMIN/57.2958
      GO TO 12
    8 SIGMA=XMIN/57.2958
      GO TO 12
    9 X=XMIN
      GO TO 12
   10 Y=XMIN
      GO TO 12
   11 Z=XMIN
   12 CONTINUE
  100 FORMAT(6F10.2,I2)
  999 FORMAT(6F10.2)
  101 FORMAT(6F10.2)
      DELTA=DELTA/57.2958
      EPS=EPS/57.2958
      SIGMA=SIGMA/57.2958
      SD=SIN(DELTA)
      S2D=SIN(2.*DELTA)
      DSQ=SD*SD
      CD=COS(DELTA)
      C2D=COS(2.*DELTA)
      DSQC=CD*CD
      SE=SIN(EPS)
      S2E=SIN(2.*EPS)
      ESQ=SE*SE
```

```
CE=COS(EPS)
C2E=COS(2.*EPS)
ESQC=CE*CE
SS=SIN(SIGMA)
S2S=SIN(2.*SIGMA)
SSQ=SS*SS
CS=COS(SIGMA)
C2S=COS(2.*SIGMA)
SSQC=CS*CS
BT(1,1)=CD*CE
BT(1,2)=CD*SE*SS-SD*CS
BT(1,3)=CD*SE*CS+SD*SS
BT(2,1)=SD*CE
BT(2,2)=SD*SE*SS+CD*CS
BT(2,3)=SD*SE*CS-CD*SS
BT(3,1)=-SE
BT(3,2)=CE*SS
BT(3,3)=CE*CS
C(1,1)=DSQC*ESQC
C(1,2)=S2D*ESQC
C(1,3)=CD*S2E
C(1,4)=DSQ*ESQC
C(1,5)=SD*S2E
C(1,6)=ESQ
C(2,1)=CD*CE*(SD*CS-CD*SE*SS)
C(2,2)=S2D*S2E*SS/2.+C2D*CE*CS
C(2,3)=SD*SE*CS+CD*C2E*SS
C(2,4)=(DSQ*S2E*SS+S2D*CE*CS)/2.
C(2,5)=SD*C2E*SS-CD*SE*CS
C(2,6)=SE*CE*SS
C(3,1)=-(DSQC*S2E*CS+S2D*CE*SS)/2.
C(3,3)=S2D*S2E*CS/2.-C2D*CE*SS
C(3,3)=CD*C2E*CS-SD*SE*SS
C(3,4)=SD*CE*(CD*SS-SD*SE*CS)
C(3,5)=SD*C2E*CS+CD*SE*SS
C(3,6)=SE*CE*CS
```

```
      C(4,1)=DSQ*SSQC+DSQC*ESQ*SSQ-S2D*SE*S2S/2.
      C(4,2)=S2D*(SSQC-ESQ*SSQ)-C2D*SE*S2S
      C(4,3)=-CD*S2E*SSQ+SD*CE*S2S
      C(4,4)=DSQC*SSQC+DSQ*ESQ*SSQ+S2D*SE*S2S/2.
      C(4,5)=-SD*S2E*SSQ-CD*CE*S2D
      C(4,6)=ESQC*SSQ
      C(5,1)=(S2D*SE*C2S+(DSQC*ESQC-C2D)*S2S)/2.
      C(5,2)=S2D*(ESQ+1.)*S2S-C2D*SE*C2S
      C(5,3)=CD*S2E*S2S/2.-SD*CE*C2S
      C(5,4)=(S2S*(DSQC-DSQ*ESQ)-S2D*SE*C2S)/2.
      C(5,5)=SD*S2E*S2S/2.+CD*CE*C2S
      C(5,6)=-ESQC*S2S/2.
      C(6,1)=DSQ*SSQ+DSQC*ESQ*SSQC+S2D*SE*SSQ/2.
      C(6,2)=S2D*(1.-(ESQ+1.)*SSQC)+C2D*SE*S2S
      C(6,3)=-CD*S2E*SSQC-SD*CE*S2S
      C(6,4)=DSQC*SSQ+DSQ*ESQ*SSQC-S2D*SE*S2S/2.
      C(6,5)=-SD*S2E*SSQC+CD*CE*S2S
      C(6,6)=ESQC*SSQC
      G(1)=(1.-DSQC*ESQC)*X*X+(1.-DSQ*ESQC)*Y*Y+ESQC*Z*Z-X*Y*S2D*ESQC+Y*
     &Z*SD*S2E+X*Z*CD*S2E
      G(2)=(DSQC*S2E*SS-S2D*CE*CS)*X*X/2.+(S2D*S2E*SS/2.+C2D*CE*CS)*X*Y+
     &(CD*C2E*SS+SD*SE*CS)*X*Z+(DSQ*S2E*SS+S2D*CE*CS)*Y*Y/2.+(SD*C2E*SS-
     &CD*SE*CS)*Y*Z-S2E*SS/2.*Z*Z
      G(3)=(DSQC*S2E*CS+S2D*CE*SS)*X*X/2.+(S2D*S2E*CS-C2D*CE*SS)*X*Y/2.+
     &(CD*C2E*CS-SD*SE*SS)*X*Z+(DSQ*S2E*CS-S2D*CE*SS)*Y*Y/2.+(SD*C2E*CS+
     &CD*SE*SS)*Y*Z-S2E*CS*Z*Z/2.
      G(4)=(DSQC*(1.-ESQ*SSQ)+DSQ*SSQ+S2D*SE*S2S)*X*X+(S2D*(SSQC+ESQ*SSQ
     &)-C2D*SE*S2S)*X*Y+(SD*CE*S2S-CD*S2E*SSQ)*X*Z+(DSQ*(1.-ESQ*SSQ)+
     &DSQC*SSQ-S2D*SE*S2S/2.)*Y*Y-(SS*S2E*SSQ+CD*CE*S2S)*Y*Z+(ESQ+ESQC*
     &SSQC)*Z*Z
      G(5)=(S2S*(DSQC*ESQ-DSQ)-S2D*SE*C2S)*X*X/2.+(C2D*SE*C2S+S2D*(ESQ+
     &1.)/2.*S2S)*X*Y+(CD*S2E*S2S/2.-SD*CE*C2S)*X*Z+(S2S*(DSQ*ESQ-DSQC)/
     &2.+S2D*SE*C2S)*Y*Y+(SD*S2E*S2S+CD*CE*C2S)*Y*Z/2.+ESQC*S2S*Z*Z/2.
      G(6)=(DSQC*(1.-ESQ*DSQC)-S2D*SE*S2S/2.+DSQ*SSQC)*X*X+(S2D*(1.-SSQC
     &*(1.+ESQ))+C2D*SE*S2S)*X*Y-(CD*S2E*SSQC+SD*CE*S2S)*X*Z+(DSQ*(1.-
     &ESQ*SSQC)+S2D*SE*S2S/2.+DSQC*SSQC)*Y*Y+(CD*CE*S2S-SD*S2E*SSQC)*Y*Z
```

```
     &+(ESQ+ESQC*SSQ)*Z*Z
      DO 1 I=1,6
      X2(I)=0.
      DO 1 J=1,6
    1 X2(I)=C(I,J)*X1(J)+X2(I)
      DO 2 I=1,6
    2 X2(I)=X2(I)+G(I)
      II=II+01
      WRITE(6,101)X,Y,Z,DELTA,EPS,SIGMA
      IF(Y.EQ.0.)TOUT(II)=0.
      IF(Y.NE.0.)TOUT(II)=ATAN2(Y,X)
      XL(II)=BT(1,1)
      XM(II)=BT(2,2)
      XN(II)=BT(3,3)
      XX(II)=X2(1)
      XY(II)=X2(2)
      XZ(II)=X2(3)
      YY(II)=X2(4)
      YZ(II)=X2(5)
      ZZ(II)=X2(6)
      WRITE(6,102)X2
  102 FORMAT(1X,'I-XX=',E10.5,'I-XY=',E10.5,'I-XZ=',E10.5,'I-YY=',E10.5,
     &'I-YZ=',E10.5,'I-ZZ=',E10.5)
      WRITE(6,103)X,Y,Z,XL(II),XM(II),XN(II),DELTA,EPS,SIGMA
  103 FORMAT(1X,'XV',F10.3,'Y=',F10.3,'Z=',F10.3,'L=',F10.3,'M=',F10.3,
     &'N=',F10.3/1X,'DELTA=',F10.5,'EPS=',F10.5,'SIGMA=',F10.5)
      GO TO (14,15,16,17,18,19),IVARY
   14 DELTA=DELTA+XINC/57.2958
      TEST=DELTA
      GO TO 20
   15 EPS=EPS+XINC/57.2958
      TEST=EPS
      GO TO 20
   16 SIGMA=SIGMA+XINC/57.2958
      TEST=SIGMA
      GO TO 20
```

```
17 X=X+XINC
   TEST=X
   GO TO 20
18 Y=Y+XINC
   TEST=Y
   GO TO 20
19 Z=Z+XINC
   TEST=Z
20 IF(IVARY.LE.3)PVARY(II)=TEST-XINC/57.2958
   IF(IVARY.GT.0)PVARY(II)=TEST-XINC
   IF(TEST.LE.XMAX)GO TO 12
   IF(IGO.GT.0)GO TO 5
   CALL EXIT
   END
```

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